Romer Model; Endogenous Growth

1. Why do advanced countries exhibit sustained growth. Assume that technological progress occurs when innovators seek out innovations.

In the neo-classical model, the stock of ideas, A is assumed to change over time, separately from the production decision. For example, in the Cobb-Douglas production function we write:

1.1 $Y = AK^{\alpha}L^{1-\alpha}$

The Romer model describes how capital stock, K and labor, L_Y combine using a stock of knowledge, A, if A is assumed to represent a stock of knowledge.

1.2 $Y = K^{\alpha}(AL_Y)^{1-\alpha}$

where α is a parameter between 0 and 1. For given level of technology, A, the production function shows constant returns to scale in K and L_Y, but if A increases, then there will be increasing returns. If capital, labor, and the stock of technology all double, then output more than doubles.

We could develop the Romer model assuing that the savings rate is given exogenously (but, remember, that in the real world, the savings rate is likely to respond to the productivity of investment.)

Capital accumulation is:

1.3 $\dot{K} = s_K Y - dK$

Labor grows exponentially at a constant rate, n:

1.4 $\frac{\dot{L}}{L} = n$

(But, remember, as countries become richer and infant mortality rates fall, rates of population growth decelerate.)

In the neoclassical model, the productivity term A grows exogenously at a constant rate. In the Romer model, growth in A is endogenous. A_t is the stock of knowledge at time, t. It changes as a function of the number of innovators.

1.5 $\dot{A} = \bar{\delta} L_A$

so Labor can be used either for innovation or production. The rate of innovation might be constant or it might be positive a function of the past stock of knowledge, or, if there are diminishing returns to the application of science, it might be a decreasing function of the stock, A. Romer writes:

1.6 $\bar{\delta} = \delta A^{\phi}$

where $\phi > 0$ means that the productivity of research increases with the stock of A and f < 0 means that productivity is declining. Noticing that R&D tended to concentrate in a few central locations, Romer added a term, L^{λ}, to his model of the stock of knowledge. If $\lambda < 1$, then researchers were wasting their time re-discovering knowledge that was already known. If $\lambda > 1$, then there were complementarities (positive knowledge spillovers) in research. If $\phi > 0$, then current scientists benefit from the knowledge of earlier science.

In the Romer model, if a constant fraction of the population is employed in R&D, the model follows the neoclassical model in predicting that all per capital growth is due to technological progress.

$G_y = g_k = g_A$
Percapita output, the capital-labor ratio, and the stock of knowledge all grow at the same rate. If there is no technological progress, then there is no growth.

1.7 $g_A = \frac{\dot{A}}{A} = \delta \frac{L_A^{\lambda}}{A^{1-\Phi}}$

The growth rate of A is constant only if the numerator and denominator of this expression are growing at the same rate. Taking logs and derivatives of both sides, this requires that:

1.8 $0 = \lambda \frac{\dot{L}_A}{L_A} - (1-\Phi)\frac{\dot{A}}{A}$

Along a balanced growth path, the growth rate in the number of researchers equals the growth of population (otherwise it eventually exceeds the population.) That is, $\frac{\dot{L}_A}{L_A} = n$. Substituting this into 1.8 yields:

1.9 $g_A = \frac{\lambda n}{1-\Phi}$

This says that long run growth depends by the growth rate of innovators and the innovation production function. What does this mean? If $\lambda = 1$ and $\phi = 0$ so that the productivity of researchers is constant at δ, then the productivity of a researcher today is independent of the stock of ideas that have been discovered in the past. The production function for knowledge is:
$\dot{A} = \delta L_A$
(Notice, if the output of new knowledge is constant, at 100 new ideas per period, and unrelated to the stock and the stock of knowledge is getting larger, then the growth rate of the stock of ideas falls over time, approaching zero.
In order to generate growth, the number of new ideas must be expanding over time, for example if the number of researchers is increasing.

How does this compare with the neoclassical model? In the neoclassical model, a higher population growth rate reduces the level of percapita income along a balanced growth path. More people means that more capital is needed to keep K/L constant, but capital runs into diminishing returns.

In the Romer model, people create new innovations which are nonrivalrous, so everyone benefits.
In Romer's original model, he assumed that $\lambda = 1$ and $\phi = 1$ so that:

1.10 $\dot{A} = \delta L_A A$

and

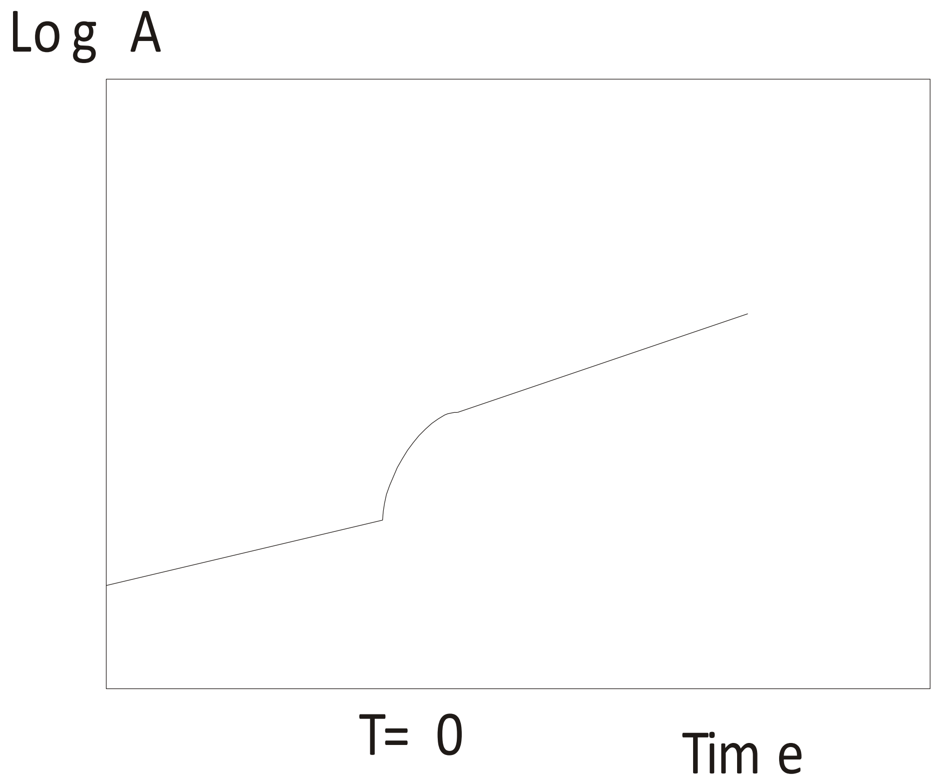
1.11 $\frac{\dot{A}}{A} = \delta L_A$

In this case, the productivity of research is proportional to the existing stock of ideas: $\bar{\delta} = \delta A$
In this form, the productivity of researchers grows over time, even if the number of researchers is constant.

Does this model fit experience? World research effort has increased enormously. This implies that the growth rate of advanced economies should also have risen rapidly, but actually US growth has been about 1.8% per year for the last 100 years, so $\phi =$ (increasing returns in research or knowledge spillovers) doesn't fit world data.

2. What happens to a permanent increase in the R&D share (assuming that $\lambda = 1$ and $\phi = 0$?) Temporarily, technological progress,

$\frac{\dot{A}}{A} = \delta L_A$ exceeds population growth, n, so the ratio L_A/A declines over time.
As this ratio declines, the rate of technological change gradually falls until the economy returns to a balanced growth path where $g_A = n$. The level of technology is permanently higher as a result of the permanent increase in R&D. There is a scale effect in levels, a larger world economy is a richer economy.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